Time Constant

We expect the temperature of a freely cooling object to fall according to:

$$T(t) = T_0 + \Delta T exp\left[-\frac{t}{\tau}\right]$$

In this equation:

- T_0 external temperature to which the object will eventually cool.
- ΔT is the initial temperature difference from T_0.
- t is the time.
- τ is the time constant.

If we differentiate this, we can find how the cooling rate will vary with time.

$$\frac{\mathrm{d}T(t)}{dt} = -\frac{\Delta T}{\tau} exp\left[-\frac{t}{\tau}\right]$$

So, when t is zero i.e. at the start of the cooling curve:

$$\frac{\mathrm{d}T(t)}{dt} = -\frac{\Delta T}{\tau}$$

So, for the house data, the initial cooling rate was around -0.95 °C/hour when the temperature difference ΔT was ~ 20 °C allows us to estimate the time constant τ as ~21 hours.

Heat Capacity

In a simple analysis, the time constant τ is given by:

$$\tau = R_{th} C$$

In this equation:

- R_{th} is the effective thermal resistance (°C/W) between the house and the external environment. This is equal to $1/HTC$ where HTC is the *heat transfer coefficient*.
- C is the heat capacity of the house (J/°C).

Knowing τ and R_{th} allows us to estimate the heat capacity C as:

$$C = \frac{\tau}{R_{th}} = \tau \times HTC$$